



MATEMÁTICA - 3º CICLO



FICHA

5



Números e Operações

Operações com números reais

Nome: _____ N.º: _____ Ano: _____ Turma: _____

Data: ____ / ____ / 20__

As propriedades das operações em \mathbb{Q} (conjunto dos números racionais) mantêm-se no conjunto \mathbb{R} :

Exemplo:

$$\sqrt[3]{27} - \sqrt{4} = 3 - 2 = 1$$

$$1 + \sqrt{50} - (\sqrt{9} - \sqrt[3]{8}) = 1 + 5\sqrt{2} - (3 - 2) = 1 + 5\sqrt{2} - 1 = 5\sqrt{2}$$

$$2\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} = (2 + 5 - 3)\sqrt{2} = 4\sqrt{2}$$

$$\sqrt{13 + \sqrt{16} + \sqrt[3]{27}} = \sqrt{13 + 4 + 8} = \sqrt{25} = 5$$

$$16^{\frac{1}{4}} = \sqrt[4]{16} = \sqrt[4]{2^4} = 2$$

Cálculo Auxiliar:

50	2
25	5
5	5
1	

Atenção:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$



Exercícios:

1. Determina

(a) $20 - (-45) : (-3)^2 + (-2) \cdot (-1)^3$

(b) $-(-2)^3 + (-1)^6 + (-1)^0 - \sqrt{25 - 3^2} - 5^2 : 25$

(c) $\frac{-(-2)^2 - \sqrt[3]{27}}{(-3 + 5)^0 - 2}$

(d) $2^{-1} + 6 \cdot \left(\frac{2}{3}\right)^{-2} - \left(-\frac{1}{3}\right)^{-1}$

(e) $\left(\frac{1}{2}\right)^{-4} : \frac{1}{2} \cdot (4^{-1})^2 + \left(-\frac{1}{6}\right)^0$

(f) $\frac{256 \cdot 4^9}{8^7}$

(g) $\frac{125^6 \cdot 25^{-3}}{(5^2)^{-3} \cdot 25^7}$

(h) $\sqrt[3]{-1}$

(i) $-\sqrt[3]{8} + 16^{\frac{1}{4}} - (-2) + 27^{\frac{1}{3}}$



2. Simplifica as expressões:

(a) $\sqrt{80} + \sqrt{20}$

(b) $\sqrt[5]{16 \cdot \sqrt[4]{18 - \sqrt[3]{5 + \sqrt{9}}}}$

Para se simplificar : **a** $(\sqrt{5})^2$ e **b** $\left(\frac{1}{\sqrt{5}}\right)^2$, faz-se da seguinte forma:

a $(\sqrt{5})^2$
 $= \sqrt{5} \times \sqrt{5}$
 $= 5$

b $\left(\frac{1}{\sqrt{5}}\right)^2$
 $= \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$
 $= \frac{1}{5}$



Exercícios:

3. Simplifica:

a $(\sqrt{7})^2$

b $(\sqrt{13})^2$

c $(\sqrt{15})^2$

d $(\sqrt{24})^2$

e $\left(\frac{1}{\sqrt{3}}\right)^2$

f $\left(\frac{1}{\sqrt{11}}\right)^2$

g $\left(\frac{1}{\sqrt{17}}\right)^2$

h $\left(\frac{1}{\sqrt{23}}\right)^2$

4. Simplifica:

a $(\sqrt[3]{2})^3$

b $(\sqrt[3]{-5})^3$

c $\left(\frac{1}{\sqrt[3]{5}}\right)^3$

5. Simplifica:

a $\sqrt{24}$

b $\sqrt{50}$

c $\sqrt{54}$

d $\sqrt{40}$

e $\sqrt{56}$

f $\sqrt{63}$

g $\sqrt{52}$

h $\sqrt{44}$

i $\sqrt{60}$

j $\sqrt{90}$

k $\sqrt{96}$

l $\sqrt{68}$

n $\sqrt{175}$

n $\sqrt{162}$

o $\sqrt{128}$

p $\sqrt{700}$

Adição e Subtração

Simplifica:

a $3\sqrt{2} + 4\sqrt{2}$
 $= 7\sqrt{2}$

b $5\sqrt{3} - 6\sqrt{3}$
 $= -1\sqrt{3}$
 $= -\sqrt{3}$



Exercícios:

6. Simplifica:

a $\sqrt{2} + \sqrt{2}$

b $\sqrt{2} - \sqrt{2}$

c $3\sqrt{2} - 2\sqrt{2}$

d $2\sqrt{3} - \sqrt{3}$

e $5\sqrt{7} + 2\sqrt{7}$

f $3\sqrt{5} - 6\sqrt{5}$

g $3\sqrt{2} + 4\sqrt{2} - \sqrt{2}$

h $6\sqrt{2} - 9\sqrt{2}$

i $\sqrt{5} + 7\sqrt{5}$

j $3\sqrt{2} - 5\sqrt{2} - \sqrt{2}$

k $3\sqrt{3} - \sqrt{3} + 2\sqrt{3}$

l $3\sqrt{5} + 7\sqrt{5} - 10$

7. Simplifica:

a $3\sqrt{2} + 2\sqrt{3} - \sqrt{2} + 5\sqrt{3}$

b $7\sqrt{2} - 4\sqrt{3} - 2\sqrt{2} + 3\sqrt{3}$

c $-6\sqrt{2} - 2\sqrt{3} - \sqrt{2} + 6\sqrt{3}$

d $2\sqrt{5} + 4\sqrt{2} + 9\sqrt{5} - 9\sqrt{2}$

e $3\sqrt{2} - 5\sqrt{7} - \sqrt{2} - 5\sqrt{7}$

f $3\sqrt{2} + 4\sqrt{11} + 6 - \sqrt{2} - \sqrt{11} - 3$

g $6\sqrt{6} - 2\sqrt{2} - \sqrt{2} - 5\sqrt{6} + 4$

h $5\sqrt{3} - 6\sqrt{7} - 5 + 4\sqrt{3} + \sqrt{7} - 8$

Multiplicação e Divisão

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$, para todo $a \geq 0$ e $b \geq 0$

Exemplo: Simplifica

a $\sqrt{3} \times \sqrt{2}$
 $= \sqrt{3 \times 2}$
 $= \sqrt{6}$

b $2\sqrt{5} \times 3\sqrt{2}$
 $= 2 \times 3 \times \sqrt{5} \times \sqrt{2}$
 $= 6 \times \sqrt{5 \times 2}$
 $= 6\sqrt{10}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, para todo $a \geq 0$ e $b \geq 0$

Exemplo: Simplifica

a $\frac{\sqrt{32}}{\sqrt{2}}$
 $= \sqrt{\frac{32}{2}}$
 $= \sqrt{16}$
 $= 4$

b $\frac{\sqrt{12}}{2\sqrt{3}}$
 $= \frac{1}{2} \sqrt{\frac{12}{3}}$
 $= \frac{1}{2} \sqrt{4}$
 $= \frac{1}{2} \times 2$
 $= 1$

Exemplo: Simplifica

a $\sqrt{2}(\sqrt{2} + \sqrt{3})$

b $\sqrt{3}(6 - 2\sqrt{3})$

Deve-se aplicar a **propriedade distributiva** da multiplicação em relação à adição, também utilizada em \mathbb{Q} :

$$\begin{aligned} \mathbf{a} \quad & \sqrt{2}(\sqrt{2} + \sqrt{3}) \\ &= \sqrt{2} \times \sqrt{2} + \sqrt{2} \times \sqrt{3} \\ &= 2 + \sqrt{6} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \sqrt{3}(6 - 2\sqrt{3}) \\ &= (\sqrt{3})(6 + -2\sqrt{3}) \\ &= (\sqrt{3})(6) + (\sqrt{3})(-2\sqrt{3}) \\ &= 6\sqrt{3} + -6 \\ &= 6\sqrt{3} - 6 \end{aligned}$$

Para simplificar $(3 - \sqrt{2})(4 + 2\sqrt{2})$ deves novamente utilizar a propriedade distributiva:

$$\begin{aligned} & (3 - \sqrt{2})(4 + 2\sqrt{2}) \\ &= (3 - \sqrt{2})(4) + (3 - \sqrt{2})(2\sqrt{2}) \\ &= 12 - 4\sqrt{2} + 6\sqrt{2} - 4 \\ &= 8 + 2\sqrt{2} \end{aligned}$$



Exercícios:

8. Simplifica:

$$\mathbf{a} \quad \sqrt{2} \times \sqrt{5}$$

$$\mathbf{d} \quad \sqrt{7} \times \sqrt{7}$$

$$\mathbf{g} \quad 3\sqrt{3} \times 2\sqrt{2}$$

$$\mathbf{j} \quad \sqrt{3} \times \sqrt{2} \times 2\sqrt{2}$$

$$\mathbf{b} \quad \sqrt{3} \times \sqrt{7}$$

$$\mathbf{e} \quad \sqrt{3} \times 2\sqrt{3}$$

$$\mathbf{h} \quad 2\sqrt{3} \times 3\sqrt{5}$$

$$\mathbf{k} \quad -3\sqrt{2} \times (\sqrt{2})^3$$

$$\mathbf{c} \quad \sqrt{3} \times \sqrt{11}$$

$$\mathbf{f} \quad 2\sqrt{2} \times \sqrt{5}$$

$$\mathbf{i} \quad \sqrt{2} \times \sqrt{3} \times \sqrt{5}$$

$$\mathbf{l} \quad (3\sqrt{2})^3 \times (\sqrt{3})^3$$

9. Simplifica:

$$\mathbf{a} \quad \frac{\sqrt{8}}{\sqrt{2}}$$

$$\mathbf{e} \quad \frac{\sqrt{20}}{\sqrt{5}}$$

$$\mathbf{i} \quad \frac{\sqrt{3}}{\sqrt{30}}$$

$$\mathbf{b} \quad \frac{\sqrt{2}}{\sqrt{8}}$$

$$\mathbf{f} \quad \frac{\sqrt{5}}{\sqrt{20}}$$

$$\mathbf{j} \quad \frac{\sqrt{50}}{\sqrt{2}}$$

$$\mathbf{c} \quad \frac{\sqrt{18}}{\sqrt{2}}$$

$$\mathbf{g} \quad \frac{\sqrt{27}}{\sqrt{3}}$$

$$\mathbf{k} \quad \frac{2\sqrt{6}}{\sqrt{24}}$$

$$\mathbf{d} \quad \frac{\sqrt{2}}{\sqrt{18}}$$

$$\mathbf{h} \quad \frac{\sqrt{18}}{\sqrt{3}}$$

$$\mathbf{l} \quad \frac{5\sqrt{75}}{\sqrt{3}}$$

10. Simplifica:

$$\mathbf{a} \quad \sqrt{2}(\sqrt{5} + \sqrt{2})$$

$$\mathbf{d} \quad \sqrt{3}(1 - \sqrt{3})$$

$$\mathbf{g} \quad \sqrt{11}(2\sqrt{11} - 1)$$

$$\mathbf{j} \quad 2\sqrt{3}(\sqrt{3} - \sqrt{5})$$

$$\mathbf{b} \quad \sqrt{2}(3 - \sqrt{2})$$

$$\mathbf{e} \quad \sqrt{7}(7 - \sqrt{7})$$

$$\mathbf{h} \quad \sqrt{6}(1 - 2\sqrt{6})$$

$$\mathbf{k} \quad 2\sqrt{5}(3 - \sqrt{5})$$

$$\mathbf{c} \quad \sqrt{3}(\sqrt{3} + 1)$$

$$\mathbf{f} \quad \sqrt{5}(2 - \sqrt{5})$$

$$\mathbf{i} \quad \sqrt{3}(\sqrt{3} + \sqrt{2} - 1)$$

$$\mathbf{l} \quad 3\sqrt{5}(2\sqrt{5} + \sqrt{2})$$

11. Simplifica:

a	$-\sqrt{2}(3 - \sqrt{2})$	b	$-\sqrt{2}(\sqrt{2} + \sqrt{3})$	c	$-\sqrt{2}(4 - \sqrt{2})$
d	$-\sqrt{3}(1 + \sqrt{3})$	e	$-\sqrt{3}(\sqrt{3} + 2)$	f	$-\sqrt{5}(2 + \sqrt{5})$
g	$-(\sqrt{2} + 3)$	h	$-\sqrt{5}(\sqrt{5} - 4)$	i	$-(3 - \sqrt{7})$
j	$-\sqrt{11}(2 - \sqrt{11})$	k	$-(\sqrt{3} - \sqrt{7})$	l	$-2\sqrt{2}(1 - \sqrt{2})$
m	$-3\sqrt{3}(5 - \sqrt{3})$	n	$-7\sqrt{2}(\sqrt{2} + \sqrt{6})$	o	$(-\sqrt{2})^3(3 - \sqrt{2})$

12. Simplifica:

a	$(1 + \sqrt{2})(2 + \sqrt{2})$	b	$(2 + \sqrt{3})(2 + \sqrt{3})$
c	$(\sqrt{3} + 2)(\sqrt{3} - 1)$	d	$(4 - \sqrt{2})(3 + \sqrt{2})$
e	$(1 + \sqrt{3})(1 - \sqrt{3})$	f	$(5 + \sqrt{7})(2 - \sqrt{7})$

Os casos notáveis da multiplicação: $(a + b)^2 = a^2 + 2ab + b^2$ e $(a + b)(a - b) = a^2 - b^2$, poder-te-ão ajudar na simplificação de números reais:

Exemplo: Usando $(a + b)^2 = a^2 + 2ab + b^2$, simplifica

a	$(\sqrt{3} + 2)^2$	b	$(\sqrt{3} - \sqrt{7})^2$
	$= (\sqrt{3})^2 + 2 \times \sqrt{3} \times 2 + 2^2$		$= (\sqrt{3})^2 - 2 \times \sqrt{3} \times \sqrt{7} + (\sqrt{7})^2$
	$= 3 + 4\sqrt{3} + 4$		$= 3 - 2\sqrt{21} + 7$
	$= 7 + 4\sqrt{3}$		$= 10 - 2\sqrt{21}$

Exemplo: Usando $(a + b)(a - b) = a^2 - b^2$, simplifica:

a	$(3 + \sqrt{2})(3 - \sqrt{2})$	b	$(2\sqrt{3} - 5)(2\sqrt{3} + 5)$
	$= 3^2 - (\sqrt{2})^2$		$= (2\sqrt{3})^2 - 5^2$
	$= 9 - 2$		$= (4 \times 3) - 25$
	$= 7$		$= 12 - 25$
			$= -13$



Exercícios:

13. Usando os casos notáveis da multiplicação $(a + b)^2 = a^2 + 2ab + b^2$, simplifica:

a	$(1 + \sqrt{2})^2$	b	$(2 - \sqrt{3})^2$	c	$(\sqrt{3} + 2)^2$
d	$(1 + \sqrt{5})^2$	e	$(\sqrt{2} - \sqrt{3})^2$	f	$(5 - \sqrt{2})^2$
g	$(\sqrt{2} + \sqrt{7})^2$	h	$(4 - \sqrt{6})^2$	i	$(\sqrt{6} - \sqrt{2})^2$
j	$(\sqrt{5} + 2\sqrt{2})^2$	k	$(\sqrt{5} - 2\sqrt{2})^2$	l	$(6 + \sqrt{8})^2$
m	$(5\sqrt{2} - 1)^2$	n	$(3 - 2\sqrt{2})^2$	o	$(1 + 3\sqrt{2})^2$

14. Usando os casos notáveis da multiplicação, simplifica:

a $(4 + \sqrt{3})(4 - \sqrt{3})$

b $(5 - \sqrt{2})(5 + \sqrt{2})$

c $(\sqrt{5} - 2)(\sqrt{5} + 2)$

d $(\sqrt{7} + 4)(\sqrt{7} - 4)$

e $(3\sqrt{2} + 2)(3\sqrt{2} - 2)$

f $(2\sqrt{5} - 1)(2\sqrt{5} + 1)$

g $(5 - 3\sqrt{3})(5 + 3\sqrt{3})$

h $(2 - 4\sqrt{2})(2 + 4\sqrt{2})$

i $(1 + 5\sqrt{7})(1 - 5\sqrt{7})$

15. Simplifica:

1. $\sqrt{2} - 5\sqrt{2} + 3\sqrt{2}$

2. $4\sqrt{5} - 2\sqrt{7} + 3\sqrt{5} + 3\sqrt{7} - 6\sqrt{5}$

3. $(\sqrt{2} - 5)^2$

4. $\sqrt{3}(4 - \sqrt{3})$

5. $(-3\sqrt{2} + \sqrt{3})(4 - \sqrt{3})$

6. $(\sqrt{a})^2 = \sqrt{a^2}$

7. $(\sqrt{a})^2 = a$

